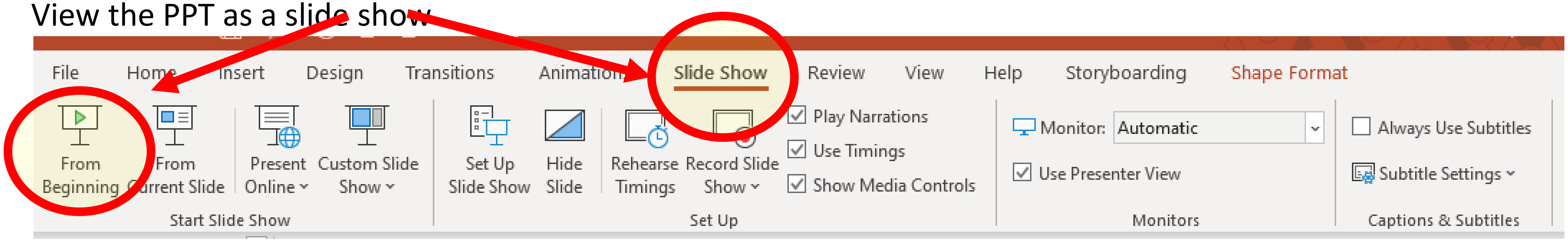
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7.2c

Graphing Other Rational Functions

Today you will:

- Graph rational functions in the form $y = \frac{ax+b}{cx+d}$.

- Practice using English to describe math processes and equations

Core Vocabulary:

- Rational function, p. 366

Previous:

- Domain
- Range
- Hyperbola
- Asymptote

Let’s look at more complicated rational functions … another common form for rational functions is

$$y = \frac{ax + b}{cx + d}$$

Important note: the equations in the numerator and in the denominator are of the same degree!

First question: what is the domain of a function in this form?

- Asked a different way, are there any limitations or illegal x values?
- Hint: is there a limitation that every fraction has?
- Answer: Yes! You cannot divide by zero.
- That means $cx + d \neq 0$ so if $x = -\frac{d}{c}$ we have problems!
- So now we know the domain: all real numbers except $x = -\frac{d}{c}$
- ***vertical asymptote*** is $x = -\frac{d}{c}$

Let’s look at more complicated rational functions … another common form for rational functions is

$$y = \frac{ax + b}{cx + d}$$

Vertical Asymptote: $x = -\frac{d}{c}$

Second question:

- What is the range?
- Best way to answer this question is to look at what happens to y as x gets *REALLY* big
- For example, let’s make up a function $\left(y = \frac{5x-1}{2x+3}\right)$ and try $x = 1{,}000{,}000$.

  $(\text{note that } a = 5, b = 2)$

$$y = \frac{5x - 1}{2x + 3} = \frac{5{,}000{,}000 - 1}{2{,}000{,}000 + 3} = \frac{4{,}999{,}999}{2{,}000{,}003} \approx \frac{5}{2} = \frac{a}{c}$$

- So as x gets really big (as $x \to \infty$) then y will approach $\frac{a}{c}$
- This gives us the ***horizontal asymptote*** $y = \frac{a}{c}$

Let's look at more complicated rational functions … another common form for rational functions is

$$y = \frac{ax + b}{cx + d}$$

Vertical Asymptote: $x = -\frac{d}{c}$

Horizontal Asymptote: $y = \frac{a}{c}$

Update – how to graph rational functions

1. Draw the asymptotes

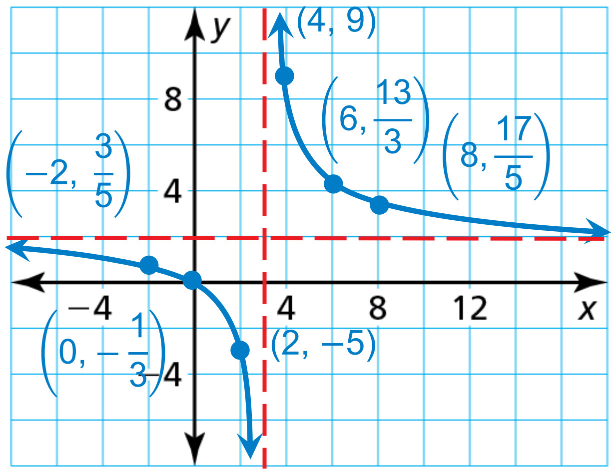
Function Form	Horizontal Asymptote	Vertical Asymptote
Translated form: $\boldsymbol{y = \frac{a}{x-h} + k}$	$y = k$	$x = h$
In $y = \frac{ax+b}{cx+d}$ form	$y = \frac{a}{c}$	$x = -\frac{d}{c}$
In simple form: $y = \frac{a}{x}$	x-axis	y-axis

2. Plot points to the left and to the right of the vertical asymptote
   - Pick numbers for x that are easy to calculate and to plot
   - If a is negative, the graph will be reflected around the x axis
3. Connect the dots
   - Draw the branches so they approach but do not touch the asymptotes

Graph $f(x) = \dfrac{2x + 1}{x - 3}$. State the domain and range.

SOLUTION

Step 1 Draw the asymptotes. Solve $x - 3 = 0$ for x to find the vertical asymptote $x = 3$. The horizontal asymptote is the line $y = \dfrac{a}{c} = \dfrac{2}{1} = 2$



Step 2 Plot points to the left of the vertical asymptote, such as $(2, -5)$, $\left(0, -\dfrac{1}{3}\right)$, and $\left(-2, \dfrac{3}{5}\right)$. Plot points to the right of the vertical asymptote, such as $(4, 9)$, $\left(6, \dfrac{13}{3}\right)$, and $\left(8, \dfrac{17}{5}\right)$.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

The domain is all real numbers except 3 and the range is all real numbers except 2.

Review/Recap

We now have 3 forms for Rational Functions:

Function Form	Horizontal Asymptote	Vertical Asymptote
Translated form: $\boldsymbol{y = \frac{a}{x-h} + k}$	$y = k$	$x = h$
In $y = \frac{ax+b}{cx+d}$ form	$y = \frac{a}{c}$	$x = -\frac{d}{c}$
In simple form: $y = \frac{a}{x}$	x-axis	y-axis

Steps for graphing Rational Functions:

1. Draw the asymptotes
2. Plot points to the left and to the right of the vertical asymptote
   - Pick numbers for x that are easy to calculate and to plot
   - If a is negative, the graph will be reflected around the x-axis
3. Connect the dots
   - Draw the branches so they approach but do not touch the asymptotes

Homework

Pg 371, #25-32